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COMMENT

On the critical dynamics of one-dimensional Potts models

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Abstract. Simple physical arguments about the movement of domain walls are used to determine the dependence of the dynamical critical exponent z on the transition rates for the one-dimensional q-state Potts model.

It is known that all one-dimensional kinetic Ising models do not belong to the same universality class. In the case of a non-conserved order parameter the value of the critical exponent z, describing how the relaxation time behaves in terms of the correlation length, $\tau \sim \xi^z$, depends on the choice of the transition rate (Deker and Haake 1979, Haake and Thol 1980). Recently a similar situation arose for the q-state Potts models. Using one transition rate Lage (1985a) first showed by a variational principle argument that $z \geqslant 3$ for $q > 2$. In another publication (Lage 1985b) he argued, based on a real space renormalisation group approach, that in fact $z = 3$ for $q > 2$. On the other hand Forgacs *et al* (1980) found $z = 2$ for all values of q, using a different transition rate.

The aim of this comment is to show that these results can be easily deduced by a straightforward generalisation of a simple physical argument devised for the Ising case by Cordery *et al* (1981). The results are confirmed by Monte Carlo simulations. Moreover the case of a conserved order parameter is treated similarly leading to $z = 5$ independently of q.

Let us recall briefly the argument of Cordery *et al* (1981) for the case of a non-conserved order parameter. They argue that the behaviour of the relaxation time near the transition temperature is determined by the time it takes for a domain wall to move at a distance ξ. Assume that the rate at which a domain wall moves by one step is ω. Then since by random walk arguments the wall must make on the average ξ^2 such steps to cover a distance ξ it follows that τ behaves like $\xi^2/\omega \sim \xi^z$. If the chosen mechanism of the motion of the wall is the fastest possible then the resulting value of z should be a least upper bound to the exact one. We can now apply this idea to the q-state Potts model defined by the reduced Hamiltonian

$$H = K \sum_i \delta_{q_i, q_{i+1}} \qquad q_i = 1, \ldots, q \qquad K = J/kT.$$

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For this model the correlation length is given by

$$\xi^{-1} = -\log \frac{\exp(K) - 1}{\exp(K) + (q-1)}.$$

We now consider two different transition rates.

(a) The transition rate chosen by Forgacs *et al* (1980):

$$W(q_i \to q_i') = \Gamma \exp[\tfrac{1}{2}(\Delta E)] \tag{1}$$

where

$$\Delta E = K(\delta_{q_{i-1},q_i'} + \delta_{q_i',q_{i+1}} - \delta_{q_{i-1},q_i} - \delta_{q_i,q_{i+1}}).$$

Since the energy difference between the initial and the final state for the spins at a domain wall is zero the rate ω behaves like Γ at all temperatures. It is easy to see that this is the fastest mechanism for the decay of the wall and hence $z = 2$ for all values of q, as found by Forgacs *et al* (1980).

(b) The transition rate chosen by Lage (1985b):

$$W(q_i \to q_i') = \Gamma \frac{\exp[-h(q_i)]}{\Sigma_{p_i=1}^{p_i=q} \exp[-h(p_i)]} \tag{2}$$

where

$$h(q_i) = K(\delta_{q_{i-1},q_i} + \delta_{q_i,q_{i+1}}).$$

For $q = 2$ both transition rates reduce essentially to Glauber's ansatz yielding $z = 2$. However for general q Lage's rate is given at the wall boundary by

$$\omega = \Gamma \frac{\exp(-K)}{[2\exp(-K) + (q-2)]}.$$

For low temperatures this goes like $\exp(-K)$ for $q > 2$. On the other hand $\xi \sim \exp(K)$. Hence $\tau \sim \xi^3$, i.e. $z = 3$.

These values of z have been checked against Monte Carlo simulations on a chain of 108 spins with periodic boundary conditions. In the initial state all Potts variables were set equal to 1. The relaxation of the order parameter was computed. For each temperature averages were taken over about 700 samples. The results are the following.

For the usual Metropolis rate $z = 1.89 \mp 0.18$ for $q = 2$ and $z = 1.93 \mp 0.15$ for $q = 3$. Note that in this scheme the transition rate is again a function of the energy difference of the initial and final state and hence a constant at a domain wall as in case (a) above.

For case (b) we find $z = 2.00 \mp 0.25$ for $q = 2$ and $z = 2.88 \mp 0.24$ for $q = 3$.

Another interesting case to consider is the one of a conserved order parameter (Kawasaki 1966). In this case two nearest-neighbour spins are exchanged. Cordery *et al* (1981) argue that since domain walls do not move independently one has to consider the motion of spins. First the spins at a wall exchange with a rate ω. Then the spin moves through the domain and comes out on the other side. For a domain of length ξ they show that this happens with probability $P_\xi = 1/\xi$. In this process the whole domain has moved one space in a time $(P_\xi \omega)^{-1} = \xi\omega^{-1}$. This must happen ξ^2 times as before to destroy the wall and thus $\xi^z \sim \xi^3/\omega$.

We again consider two cases.

(a) The rate chosen by Kawasaki:

$$W(q_i q_{i+1} \to q_{i+1} q_i) = \Gamma(1 - \delta_{q_i q_{i+1}})\Omega \tag{3}$$

with

$$\Omega = 1 + \tanh[\tfrac{1}{2}(h(q_{i+1}q_i) - h(q_iq_{i+1}))]$$

where

$$h(q_iq_{i+1}) = K(\delta_{q_{i-1},q_i} + \delta_{q_i,q_{i+1}} + \delta_{q_{i+1},q_{i+2}})$$

and correspondingly for $h(q_{i+1}q_i)$. In this case $\omega = 2\Gamma(1 - \tanh(K))$ at a domain wall and hence $\omega \sim \xi^{-2}$ for low temperatures yielding $z = 5$ for all values of q. This value has been obtained for the Ising case by Zwerger (1981).

(b) Extension of Lage's choice:

$$W(q_iq_{i+1} \to q_{i+1}q_i) = \Gamma(1 - \delta_{q_i,q_{i+1}})\Omega \tag{4}$$

with

$$\Omega = \frac{\exp(-h(q_i, q_{i+1}))}{\sum_{q_i=1}^{q_i=q} \sum_{q_{i+1}=1}^{q_{i+1}=q} \exp(-h(q_i, q_{i+1}))}.$$

Since the final state for an exchange at a domain wall has zero energy this choice leads to the same asymptotic behaviour as before and hence $z = 5$ for all q.

In conclusion we see that the study of interface dynamics, using simple physical arguments, for the one-dimensional q-state Potts model leads to a straightforward determination of the dynamical critical exponent z for different transition rates.

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